$$aA_3 \equiv \int_0^{s_3} A ds,\ bA_3 = -\int_0^{s_3} AU ds,$$

we obtain

$$k = (1/2a)(\sqrt{1 - 4ba} - 1). \quad \text{(A5)}$$

It follows from (A5) that $k^2 \sim k^2(M)$ for $k(M) \ll 1/4$, and that $k^2 \sim k(M)$ for $k(M) \gg 1/4$. We note that $U_0(M) = -2M(M-1)^2/(2-M)^2(2M-1)$ is not small for all considered Mach numbers M and the well cannot be regarded as shallow.

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CALCULATION OF TOROIDAL INDUCTIVE ACCUMULATORS WITH A D-CROSS SECTION FROM PARAMETERS OF A DISCHARGE PULSE AND OF THE CHARGING DEVICE

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One method of increasing the power of the discharge pulse of an inductive accumulator is based on the use of a scheme with a switch-over: With charging, the sections of the accumulator are connected in series and with discharge, in parallel. In this case, rigid requirements are imposed on the symmetry of the cross section. Such requirements are satisfied by constructions with a toroidal field (another of their advantages is the absence of scattering fields). In [1] it is shown that, for windings on a thin busbar, with a width increasing proportionally to the radius (an s-coil), for a thin conductor of constant width [2] (an l-coil), there exists a profile with which the pressure of the toroidal field is balanced by the voltage of the curved part of the coil. The internal rectilinear section of the coil is subject to compression in the direction of the principal axis and to longitudinal elongation. In a construction with such a profile, with uniform equilization of the radial compression, the action of the bending moments is everywhere completely excluded. The form of the profile, with which the winding does not undergo the action of the bending moments, is called a D-section. A construction with a D-section is optimal with respect to mechanical strength. In [1] it is shown that such a construction is also optimal with respect to energy capacity. Therefore, in comparison with other variants of toroidal constructions in coils with a D-section, the specific parameters are found to be the highest.

1. Method of Calculation

The form and the dimensions are determined by two parameters: the mean radius $r_0 = (r_1 +$

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$r_2)/2$ and the toroidal character $\varepsilon = (r_2 - r_1)/2r_0$ (r_1, r_2 are the internal and maximal external radii of the coil). In addition, any real construction has a turn of finite thickness. Here it is postulated that the thickness of a turn d is everywhere constant, and small in comparison with a given characteristic dimension. In this case, the coefficient of the stress is obtained by multiplication of the permissible elongational stress of the material of the coil σ by its thickness d. Thus, for calculation of the constructional dimensions and weight of the winding of a toroidal inductive accumulator with a D-section, we must dispose of three independent relationships, connecting r_0, ε, and d with given electrical and energy parameters. For definiteness, we shall consider here the discharge of an inductive accumulator with breaking of each turn of the winding.

There are assumed to be given: the energy of the inductive accumulator W_0, the maximal power of a discharge pulse P_a, the maximal value of the breaking voltage V_a, and the parameters of the charging device. If an electric transmission line is used for charging the inductive accumulator, the sole parameter of the charging device is its power. Another source of the charge can be an electrical machine assembly, consisting of a turbogenerator with a flywheel (the connection with the inductive accumulator is effected through a transformer—rectification device).

In this case, in addition to the maximal power P_0, there must be given the maximal energy of rotation K_0 and the energy of rotation K_f which remain in the electric machine assembly at the end of a charging cycle. For a charging device and for the discharge pulse, by the maximal power there is understood its reactive value, i.e., the product of the maximal values of the current and the voltage.

For convenience, in accordance with the adopted definition of P_a we introduce the notation

$$Y = NI_0 = P_a/V_a$$

(N is the number of turns; I_0 is the charging current) and the dimensionless variables $x = r/r_0$ and $\zeta = z/r_0$. In the calculation, we use

a) the condition of the equilibrium of the curved section of a turn

$$p_H = \sigma d(k_1 + k_2), \tag{1.1}$$

where p_H is the effective magnetic pressure, and

$$k_1 = \frac{\zeta''}{r_0(1+\zeta'^2)^{3/2}}, \quad k_2 = \frac{1}{r_0}\frac{\zeta'}{(1+\zeta'^2)^{1/2}} \tag{1.2}$$

are the principal curvatures of the axisymmetric shell (a prime means differentiation with respect to x); substitution of (1.2) into (1.1) gives a differential equation with the boundary conditions

$$\begin{aligned} &\text{with } x = 1-\varepsilon,\ \zeta' = +\infty, \\ &\text{with } x = 1+\varepsilon,\ \zeta' = -\infty; \end{aligned} \tag{1.3}$$

b) an expression for the energy of the inductive accumulator

$$W_0 = \frac{2Y^2}{c^2} r_0 \int\limits_{1-\varepsilon}^{1+\varepsilon} \frac{\zeta(x)}{x}\, dx; \tag{1.4}$$

c) a relationship determining the value of the active resistance of the winding to a direct current under charging conditions, taking account of the data on the power and the voltage for a discharge pulse.

This relationship is derived in the following manner. The expression for the resistance of the coil, neglecting the slight inhomogeneity of the distribution of the current density over the cross section of a turn, can be written in the form

$$R = \frac{\rho Y^2}{\pi d} \Phi(\varepsilon),$$

where ρ is the specific resistance of the material of the winding; the value of the function Φ(ε) depends only on the toroidal character and the type of the winding. On the other hand, with charging by an electric machine assembly,

$$R = gP_0/I_0^2.$$

The coefficient g depends on the parameters K_0, K_f, W_0, and is determined by a simple calculation from the solution of a differential equation of the second order with constant coefficients describing the charging process. From the equality of the two latter expressions, we obtain

$$(\rho Y^2/\pi d)\Phi(\varepsilon) = gP_0. \tag{1.5}$$

Calculation of the coil thus reduces to the solution of a system of three equations (1.3), (1.4), and (1.5) with the three unknowns r_0, ε, and d.

For representation of the solution in compact form, we introduce the following three parameters with the dimensionality of length:

$$a_1 = \frac{\rho Y^2}{\pi g P_0}, \qquad a_2 = \frac{c^2 W_0}{2Y^2}, \qquad a_3 = \frac{Y}{2c} \frac{1}{(\pi\sigma)^{1/2}}. \tag{1.6}$$

2. s-Coil

It is postulated that the turns are rigidly interconnected and that the construction can be regarded as a shell with two curvatures. The equilibrium equation (1.1) can be written in the following fashion:

$$\frac{1}{2\pi r_0^2}\left(\frac{Y}{cx}\right)^2 = \sigma d\,(k_1 + k_2),$$

where k_1, k_2 are determined by formulas (1.2). The solution of the system (1.3)-(1.5) in this case can be represented in the form

$$d = a_1 \Phi_s(\varepsilon); \tag{2.1}$$

$$r_0 = \frac{a_2}{F_s(\varepsilon)}; \tag{2.2}$$

$$\frac{a_1 a_2}{a_3^2} = \chi_s(\varepsilon). \tag{2.3}$$

Here

$$\Phi_s(\varepsilon) = \int_{1-\varepsilon}^{1+\varepsilon} \left[1 + \left(\frac{d\zeta}{dx}\right)_s^2\right]^{1/2} \frac{dx}{x} + \frac{\zeta_{1s}}{1-\varepsilon}, \qquad F_s(\varepsilon) = \int_{1+\varepsilon}^{1-\varepsilon} \left(\frac{d\zeta}{dx}\right)_s \ln x\, dx + \zeta_{1s} \ln \frac{1}{1-\varepsilon},$$

$$\left(\frac{d\zeta}{dx}\right)_s = \frac{\frac{1-\varepsilon}{2} \ln \frac{1+\varepsilon}{1-\varepsilon} - \ln \frac{x}{1-\varepsilon}}{\left[\frac{x^2}{4}\left(\ln \frac{1+\varepsilon}{1-\varepsilon}\right)^2 - \left(\frac{1-\varepsilon}{2} \ln \frac{1+\varepsilon}{1-\varepsilon} - \ln \frac{x}{1-\varepsilon}\right)^2\right]^{1/2}}, \qquad \zeta_{1s} = \int_{1+\varepsilon}^{1-\varepsilon} \left(\frac{d\zeta}{dx}\right)_s dx, \quad \chi_s(\varepsilon) = \ln \frac{1+\varepsilon}{1-\varepsilon} \frac{F_s(\varepsilon)}{\Phi_s(\varepsilon)}$$

with x = 1 + ε, ζ = 0.

The maximal energy density and the volume of the material of the winding are calculated using the formulas

$$\left(\frac{H^2}{8\pi}\right)_{s\,\max} = \frac{\Upsilon^2}{2\pi c^2 a_2^2}\,\frac{F_s^2(\varepsilon)}{(1-\varepsilon)^2}; \tag{2.4}$$

$$V_s = 4\pi a_1 a_2^2\,\frac{\Phi_s(\varepsilon)\,G_s(\varepsilon)}{F_s^2(\varepsilon)}, \tag{2.5}$$

$$G_s(\varepsilon) = \int_{1-\varepsilon}^{1+\varepsilon} x\left[1+\left(\frac{d\zeta}{dx}\right)_s^2\right]^{1/2} dx + \zeta_{1s}(1-\varepsilon).$$

3. l-Coil

In this case, the pressure of the field at the surface, rising proportionally to the radius, is compensated by the voltage of a turn of constant width only across the lines of force; therefore, Eq. (1.1) assumes the form

$$\frac{1}{2\pi(1-\varepsilon)\,r_0^2\,x}\left(\frac{\Upsilon}{c}\right)^2 = \sigma d k_1.$$

Solving the system of equations for an l-coil, we obtain

$$d = a_1\Phi_l(\varepsilon); \tag{3.1}$$

$$r_0 = \frac{a_2}{F_l(\varepsilon)}; \tag{3.2}$$

$$\frac{a_1 a_2}{a_3^2} = \chi_l(\varepsilon), \tag{3.3}$$

where

$$\Phi_l(\varepsilon) = \frac{1}{1-\varepsilon}\left\{\int_{1-\varepsilon}^{1+\varepsilon}\left[1+\left(\frac{d\zeta}{dx}\right)_l^2\right]^{1/2} dx + \zeta_{1l}\right\},$$

$$F_l(\varepsilon) = \int_{1+\varepsilon}^{1-\varepsilon}\left(\frac{d\zeta}{dx}\right)_l \ln x\, dx + \zeta_{1l}\ln\frac{1}{1-\varepsilon},$$

$$\chi_l(\varepsilon) = \frac{1}{1-\varepsilon}\ln\frac{1+\varepsilon}{1-\varepsilon}\,\frac{F_l(\varepsilon)}{\Phi_l(\varepsilon)},$$

$$\left(\frac{d\zeta}{dx}\right)_l = \frac{\frac{1}{2}\ln\frac{1+\varepsilon}{1-\varepsilon} - \ln\frac{x}{1-\varepsilon}}{\left[\left(\frac{1}{2}\ln\frac{1+\varepsilon}{1-\varepsilon}\right)^2 - \left(\ln\frac{x}{1-\varepsilon} - \frac{1}{2}\ln\frac{1+\varepsilon}{1-\varepsilon}\right)^2\right]^{1/2}},$$

$$\zeta_{1l} = \int_{1+\varepsilon}^{1-\varepsilon}\left(\frac{d\zeta}{dx}\right)_l dx,$$

with $x = 1+\varepsilon$, $\zeta = 0$.

The formulas for the maximal energy density and the volume of the material of the winding are written in the form

$$\left(\frac{H^2}{8\pi}\right)_{l\,\max} = \frac{\Upsilon^2}{2\pi c^2 a_2^2}\,\frac{F_l^2(\varepsilon)}{(1-\varepsilon)^2}; \tag{3.4}$$

$$V_l = \frac{4\pi a_3^4}{a_1}\left(\ln\frac{1+\varepsilon}{1-\varepsilon}\right)^2. \tag{3.5}$$

4. sl-Coil

A construction with turns expanding proportionally to the radius (as in an s-coil), but without a rigid connection between the turns (as in an l-coil), we call an sl-coil. In this case the equilibrium equation

$$\frac{1}{2\pi r_0^2}\left(\frac{Y}{cx}\right)^2 = \sigma d k_1$$

is completely solved analytically (in the preceding cases, an analytical expression can be obtained only for the first integral).

The solution of the system has the form

$$d = a_1 \Phi_{sl} \varepsilon ; \tag{4.1}$$

$$r_0 = \frac{a_2}{F_{sl}(\varepsilon)}; \tag{4.2}$$

$$\frac{a_1 a_2}{a_3^2} = \chi_{sl}(\varepsilon), \tag{4.3}$$

where

$$\Phi_{sl}(\varepsilon) = \frac{\pi\varepsilon}{\sqrt{1-\varepsilon^2}}\frac{1}{1-\varepsilon},$$

$$F_{sl}(\varepsilon) = \int_{1+\varepsilon}^{1-\varepsilon}\left(\frac{d\zeta}{dx}\right)_{sl}\ln x\,dx + \zeta_{1sl}\ln\frac{1}{1-\varepsilon},$$

$$\chi_{sl}(\varepsilon) = \frac{2\varepsilon}{1-\varepsilon^2}\frac{F_{sl}(\varepsilon)}{\Phi_{sl}(\varepsilon)}, \tag{4.4}$$

$$\left(\frac{d\zeta}{dx}\right)_{sl} = \frac{\left(\frac{1-\varepsilon^2}{x}-1\right)\frac{1}{\varepsilon}}{\left[1-\frac{1}{\varepsilon^2}\left(\frac{1-\varepsilon^2}{x}-1\right)^2\right]^{1/2}}, \quad \zeta_{1ls} = \frac{\varepsilon^2\pi}{(1-\varepsilon^2)}.$$

The volume of material of the winding

$$V_{sl} = 4\pi a_1 a_2^2 \frac{\Phi_{sl}(\varepsilon)}{F_{sl}^2(\varepsilon)}\frac{\pi\varepsilon}{\sqrt{1-\varepsilon^2}}\left(1+\varepsilon-\frac{\varepsilon^2}{2}\right).$$

The formula for determining the energy density in an sl-coil is analogous to (2.4).

5. Results of Calculations

The most labor-consuming part of the calculation consists in calculating the dependence of the functions F_s, Φ_s, G_s, F_l, Φ_l, and F_{sl} on ε, determined by complex integrals with singularities of the functions under the integral signs at the upper and lower limits. The functions and their combinations were calculated on an electronic computer. The interval of values of ε for an s-coil is limited by the limiting value of the toroidal character $\varepsilon_* = 0.564 \ldots$, and for l- and sl-coils, $\varepsilon_* = 1$.

The results needed for calculation of s-coils are given in Fig. 1. Figure 2 shows a curve of the function which is used to determine the weight of the winding.

Results relating to calculation of l- and sl-coils are given in Figs. 3 and 4.

The construction is calculated in the following manner. Formulas (1.6) are used to find a_1, a_2, a_3 and the ratio $a_1 a_2/a_3^2$, equal to the value of the functions $\chi_s(\varepsilon)$ (2.3), $\chi_l(\varepsilon)$ (3.3), or $\chi_{sl}(\varepsilon)$ (4.3). The curves of these functions (see Figs. 1, 3, and 4) are used to find the corresponding values of ε. From the known values of ε, returning to Figs. 1-4, and using formula (4.4), we find the numerical values of all the necessary functions, which are then substituted into (2.1), (2.2), (2.4), and (2.5) for calculation of an s-coil, into (3.1), (3.2),

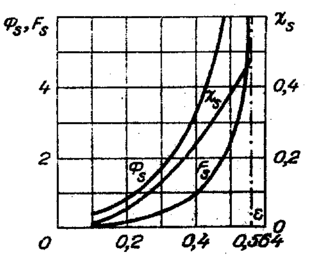

Fig. 1

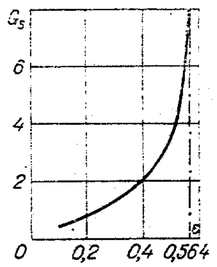

Fig. 2

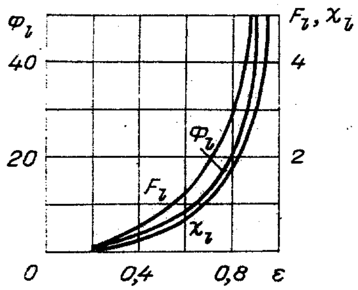

Fig. 3

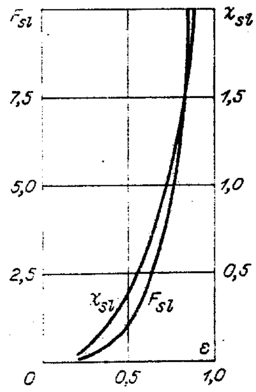

Fig. 4

(3.4), and (3.5) for an l-coil, or into (4.1)-(4.3), and (2.4) if an sl-coil is being calculated.

During the process of the calculations it may be found that the relative thickness of the winding is too great and cannot be calculated like a thin winding. In this case, the dimensions of toroidal constructions can be brought into correspondence with the starting assumptions if the value of ε is lowered, for which the value of the function $\chi(\varepsilon)$ must be decreased. If the ratio $a_1 a_2/a_3^2$ is expressed in terms of the parameters of the electric-discharge system, it is then found that

$$\chi(\varepsilon) = 2c^4\rho\sigma W_0/gP_0Y^2.$$

While it is impossible to decrease W_0, it is possible to calculate a construction with an increased reserve of strength, giving lower values of σ. Still another possibility is connected with an increase in P_0. However, in this case it is advisable to change the method of calculation, giving, as one of the starting parameters, the coefficient of the energy losses with a rapid leadout.

6. Calculation of Toroidal Inductive Accumulators from a Given Coefficient of the Losses

In a toroidal inductive accumulator, the energy included in a volume within which the magnetic lines of force penetrate the turns, with a duration of a discharge pulse which is short in comparison with the time of diffusion to a depth d, is dissipated by eddy currents. The ratio of this energy to W_0 is determined by the coefficient of the losses α. It can be shown that for a coil with a thin winding, in the first approximation $\alpha = d/r_0\varepsilon$.

As an example, we give the solution of the starting equations with the calculation of an l-coil for the case where the number of given parameters excludes P_0, which is replaced by the parameter α (here the parameter a_1, depending on P_0, is determined as the result of a calculation):

$$r_0 = a_2/F_l(\varepsilon); \quad (6.1)$$

$$d = \alpha a_2\varepsilon/F_l(\varepsilon); \quad (6.2)$$

$$\frac{\alpha a_2^2}{a_3^2} = \frac{1}{\varepsilon(1-\varepsilon)} \ln\frac{1+\varepsilon}{1-\varepsilon} F_l^2(\varepsilon); \quad (6.3)$$

$$a_1 = \alpha a_2\varepsilon/F_l(\varepsilon)\Phi_l(\varepsilon). \quad (6.4)$$

The value of ε is determined using (6.3). Further, the values of r_0, d, a_1 are determined using formulas (6.1), (6.2), and (6.4). Substituting a_1 in (1.6) yields the value of P_0. s- and sl-coils are calculated in analogous fashion.

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CALCULATION OF THERMAL STRESSES ARISING IN ELECTRICALLY CONDUCTING MATERIALS WITH THE PASSAGE OF A HIGH-CURRENT PULSE

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A high-current pulse, passing through a rod, whose duration is less than the period of the free vibrations of the rod, sets up thermoelastic vibrations in the rod [1, 2]. A calculation of the stresses arising in a vibrating rod was made in [2], taking account of the temperature dependence of the elastic modulus, under the assumption of a homogeneous distribution of the current.

Neglect of the skin effect in a conductor is not always justified. If the radius of the rod exceeds the thickness of the skin depth, the inhomogeneity of the distribution of the current density must be taken into consideration [3]. An inhomogeneous distribution of the current in a conductor with a high-frequency electrical pulse sets up a temperature gradient and brings about thermal stresses in the material of the conductor [4].

In the present work a calculation is made of the thermal stresses in a sample, subjected to heating by a current pulse, taking account of the skin effect; the statement of the electrodynamic problem coincides with the statement of the problem in [3]. For a description of the process of pulsed heating, the equations of electrodynamics are supplemented by the equations of thermal conductivity and elasticity [5]

$$c_V \frac{\partial T}{\partial t} + \frac{E}{1-2\mu}\frac{\alpha T}{3}\frac{\partial}{\partial t}\frac{\partial u_l}{\partial x_l} = \frac{\partial}{\partial x_l}\left(\varkappa \frac{\partial T}{\partial x_l}\right) + Q(x, t); \quad (1)$$

$$\rho \frac{\partial^2 u_l}{\partial t^2} = \frac{\partial \tau_{lm}}{\partial x_m} + F_l, \quad l, m = 1, 2, 3, \quad (2)$$

where T is the temperature; c_V is the specific heat capacity; E is the Young modulus; μ is the Poisson coefficient; α is the coefficient of volumetric expansion; Q is a function, describing the Joule heating of the sample; τ_{lm} is the tensor of the internal stresses; u_l are the components of the vector of the deformation; ρ is the density; F_l is the component of the volumetric forces acting on the sample.

We consider the quasi-steady-state electrodynamic problem with a constant conductivity σ. The condition of the quasistationary character of the electromagnetic field for conductors with a length of ~1 m is satisfied for pulses with a duration $\Delta t \geqslant 10^{-6}$ sec. Ohm's law is applied in its simplest form:

$$j_l = \sigma E_l,$$

where j_l is the current density; E_l is the intensity of the electrical field.

In such a statement, the electromagnetic problem is solved separately from the equations of thermal conductivity and elasticity. Under the assumption of the presence of axial symmetry of the problem, the current density has one component differing from zero, $j_z(r, t) = j(r, t)$, for which the equation has the form

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